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The research problems successfully considered during the past year can be broadly classified into four areas: (1) Communication Networks, (2) Variable Length Codes, (3) Digital Systems and Computer Applications, and (4) Topics in Pure and Applied Graph Theory. ~~We will briefly describe~~ the results obtained in each of these areas and discuss some possible extensions and the work currently in progress.

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From July 1, 1976 to June 30, 1977

Introduction

The research problems successfully considered during the past year can be broadly classified into four areas:

(1) Communication Networks,

(2) Variable Length Codes,

(3) Digital Systems and Computer Applications, and

(4) Topics in Pure and Applied Graph Theory.

We will briefly describe the results obtained in each of these areas and discuss some possible extensions and the work currently in progress.

(1) Communication Networks

Hakimi defined the notions of a p-center and a p-median of a network in two papers written over a decade age. Since then a field centered about these ideas has evolved called "location theory". In a paper by Hakimi, Pierce and Schmeichel [1], it is shown that Hakimi's original algorithm for finding a 1-center of a network may be considerably refined and the resulting algorithm requires in the order $n^4 \log n$ operations. Furthermore, if there are no weights associated with vertices of the network, then the algorithm requires $O(n^3 \log n)$ operations. This algorithm can also be used to find a least diameter tree of a weighted graph in $O(n^3 \log n)$ operations.

More recently Kariv and Hakimi [2] have dramatically improved all of the results in [1] and have shown that the general p-center problem is an

NP-complete problem and thus no polynomially bounded algorithm for its solution can be expected. In fact, Kariv and Hakimi [2] give algorithms for finding a p-center of a tree or a general network for vertex weighted or vertex unweighted cases. Every algorithm the authors give is polynomially bounded with the exception of those problems which they prove to be NP-complete.

Finally Kariv and Hakimi [3] studied the p-median problem in networks. The authors have shown that the p-median problem for a general network is also NP-complete. However, we developed an algorithm for finding p-median of a tree. The algorithm has the order of complexity $O(n^2p^2)$ where n is the number of vertices. This is an order of magnitude better than the best know algorithm due to Matula and Kolde for the same purpose.

The principal investigator was invited to give a talk on the recent development on the p-center problem [4]. More recently, the principal investigator has agreed to give an invited lecture on network location theory at an International Symposium on Locational Decisions in Calgary Canada [3]. Other topics in this area are under investigation.

(2) Variable Length Codes

A problem in communication of information involves conversion of statistically independent characters $\{x_1,x_2,x_3,\ldots,x_M\}$ produced by the information source with probabilities $\{p_1,p_2,\ldots,p_M\}$, to a set of sequences of letters from the channel alphabet, $\{a_1,a_2,\ldots,a_D\}$. More precisely the encoder associates with each source character, x_i a sequence w_i of letters from the channel alphabet, $i = 1,2,\ldots,M$. The set $C = \{w_1,w_2,\ldots,w_M\}$ is called a code. For a code to be useful, it must be uniquely decipherable, i.e. any sequence of letters from the channel alphabet can be broken down into a sequence of code words in no more than one way. A code C is called a prefix code if there does not exist integers i and j and a sequences of channel alphabet letters b such that $w_j = w_i b$. A prefix code is uniquely deciperable, but a uniquely decipherable code may not be a prefix code. There are general techniques for constructing prefix codes with a prescribed code word lengths (unless no such a code exists). Now suppose we are not merely interested in code word lengths; we may be also interested in the composition of code words. More precisely, one may want to construct to a code in which there are $N(y_1,y_2,\ldots,y_D)$ code words in which letter a_i appears y_i times for $i = 1,2,\ldots,D$. This problem is referred to as the construction of codes with given compositions. Given $N(y_1,y_2,\ldots,y_D)$ for all $y_1,y_2,\ldots,y_D$, one of this investigator's students, gave necessary and sufficient conditions for existence of a prefix code with the given prescribed compositions. The outstanding remaining problem, and one that is often conjectured, is that if for some prescribed compositions there exists no prefix code, then there is also no uniquely decipherable code with the same compositions. Ntafos and Hakimi [6] give further evidence in support of this conjecture. These authors, show that the conjecture is correct if there are only two code word lengths. This problem has application in information theory and also computer file organization.

(3) Digital Systems and Computer Applications

Our effort in this area can be divided into two parts.

(a) Fault Analysis in Digital Systems. Suppose we have k mini-computers (microprocessors) aboard an aircraft for the purposes of navigation, control, communications, monitoring, etc. These computers thus control and monitor the behavior of all other systems aboard. Let us assume on regular intervals, say once every five minutes the computers go into a "testing mode" which lasts, say 2 seconds. In this testing mode each computer is assigned to test some other computers as well as some other systems aboard. The results of the testing mode are the expressed opinions of various computers about the conditions of the other computers and systems which they have been assigned to test. Each expressed opinion has two values - for example, a computer may say a unit is either working (non-faulty) or is not working (faulty). However since we do not know (a priory) which computers are working, every "opinion" expressed is merely an opinion and not a fact. Having at hand the results of all the test which were carried out by various computers, we would like to identify with a high degree of reliability which units (if any) are faulty. The characterization of the capability of such systems for identifying faults, their natural generalizations, and the algorithm associated with them are the subjects of a number of papers by this author and his associates. These results are extremely encouraging and interesting. These ideas could lead to an alternative approach to reliable computation rather than the classical approach of (triple) Modular Redundancy. Our work in this area is being rigorously pursued. A tutorial paper by this writer on the subject appeared as a lead article in a book [7]. This writer is invited to give a lecture [8] on this subject at the ORSA/TIMS Fall 1977 meeting at Atlanta.

(b) Computer Application and Algorithms.

Two problems are under consideration.

(i) A Heuristic Approach to the Set Covering Problem. As it is well known, the set covering problem occurs in great many optimization problems in information theory, switching theory, theory of computation, and graphs and combinatorics. Also the set covering problem is a NP-complete; thus a heuristic approach is the only viable approach to it. A paper by Chwa and Hakimi [9] presents efficient algorithms for the set covering problem which give better results than existing algorithms and also are faster than almost all available ones. We have also developed computer programs based on our algorithms. A worse case analysis, presented in this paper, shows that the ratio of worst solution to the optimum solution is proportional to the log of the maximum degree. Further work on this problem is continuing. It is expected that the finding of a solution which at worst differs from the optimum by multiplicative constant is also NP-complete. If such result is proven it would be of great theoretical as well as practical significance.

(ii) Software Validation. A feasible approach to software validation involves covering a digraph representing the flow chart of a computer program, by a least number of paths. Dilworth's theorem suggests an approach to this problem if the resulting digraph is acyclyc. We have devised a technique for generalizing Dilworth's theorem to the cyclic digraphs and we have succeeded in developing an efficient algorithm for finding the least number of paths(chains) to cover any digraph. Many practically motivated variations of this problem are being considered. A paper on this subject is forthcoming

(4) Topics in Pure and Applied Graph Theory

A new class of realizability problems in graphs was considered by Patrinos and Hakimi [10]. These problems generally involve relations between a sequence of positive integer-pairs and graphs or digraphs. For example, given a sequence of positive integer pairs (α_1, β_1), (α_2, β_2), ..., (α_m, β_m), is there a graph with m edges $\{e_1, e_2, \ldots, e_m\}$ such that each edge e_i is incident at a vertex of degree α_i and a vertex of degree β_i? Solution of this problem and many of its various generalizations are presented in this paper.

We also worked on the problem of existence of planar graphs with prescribed vertex degrees. This problem is extremely complex and many papers have been written on it, but the complete solution still seems to be out of reach. A paper by Schmeichel and Hakimi [11] presents a great many new results which shed considerable light on the subject. This problem is important because of its possible relation to the four color problem. Its practical significance lies in its possible applications to printed circuit boards as well as other layout problems.

Finally Schmeichel and Hakimi [12] give a characterization of the sequences $(d_1,d_2,\ldots,d_p)$, $d_1 \geq d_2 \ldots \geq d_p$, which are vertex degrees of a graph which has a Hamiltonian path between a vertex of degree d_i and a vertex of degree d_j for any specified i and j, $1 \leq i < j \leq p$.

We are also making progress on some other theoretical problems involving degree sequences of graphs with designated properties. Although these results may not have immediate applications, they help fill out the existing gaps in present theory which makes the theory more useful for future applications.

Hakimi and Schmeichel [13], present a theorem which, in a sense, gives the best possible result on the connectivity of the maximal planar graphs. This result generalized Whitney's classical results on this subject and has possible application to the theory of reliable planar networks.

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